Algebra (astratta)

venerdì 23 agosto 2024 11:10

Gruppi → Burnside

↓

gruppo X = insieme di elementi

↓

con operazioni

$(\mathbb{Z}, +)$

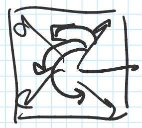
$1, 2, 3, 4, \dots \quad -1, -2, -4 \dots$

- gruppo

1 - $\forall x, y \in G \Rightarrow x \oplus y \in G$

2 - $\forall x \Rightarrow \exists x^{-1} \Rightarrow x \oplus x^{-1} = 1\!\!1_G$

3 - $\exists 1\!\!1_G$



G gruppo finito → X finito

$\mathbb{Z}_5 \rightarrow ([0], [1], [2], [3], [4])$

$\underbrace{0 \quad 1 \quad 2 \quad 3 \quad 4}$

no zero

$(\mathbb{Z}_5, +) \rightarrow (\mathbb{Z}_5^*, \cdot)$

$(1, 2, 3, 4)$

$a^{p-1} \equiv_p 1 \qquad (a, p) = 1$

$a^4 \equiv_5 1$

a

$$f : G \to G$$
$$x \mapsto a x$$

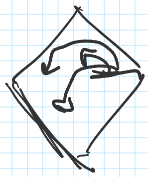
l'insieme delle immagini si chiama ORBITA

$$\underbrace{a \to a^2 \to a^3 \to a^4 \to a^5}$$

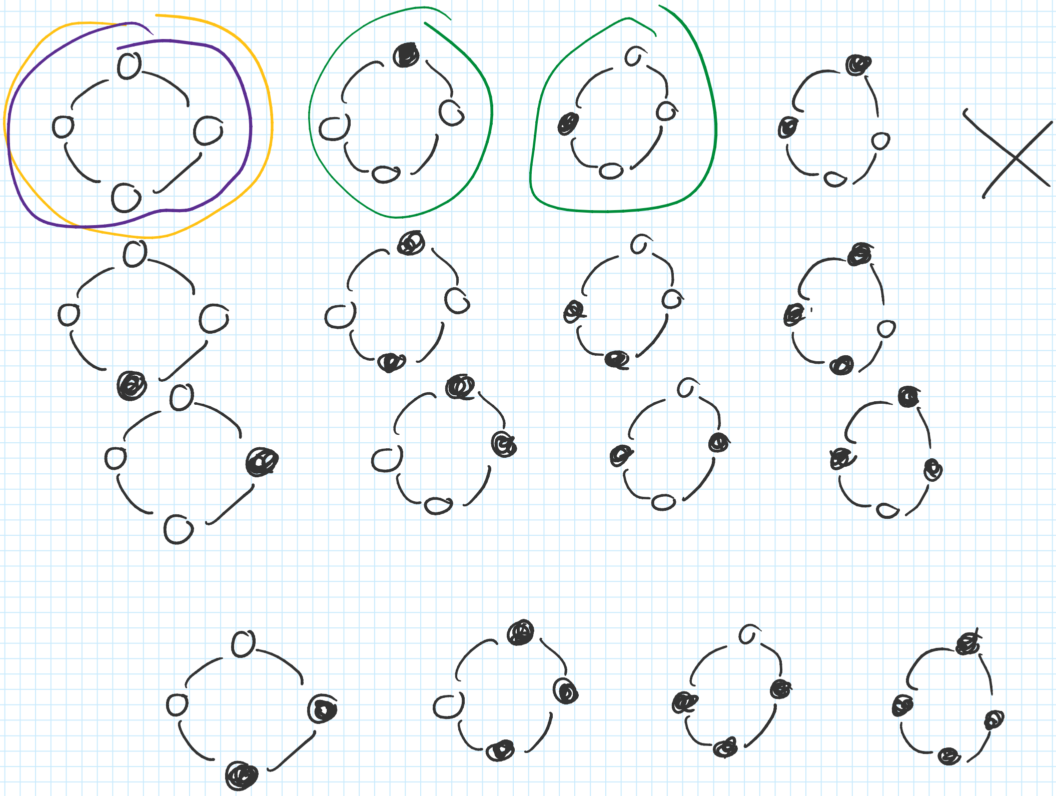
$$a = a^5$$

$$\underbrace{a^{-1} \cdot a}_{=1} = \underbrace{a^{-1} \cdot a^5}_{=a^4}$$

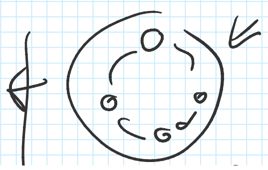
$$1 = a^4$$

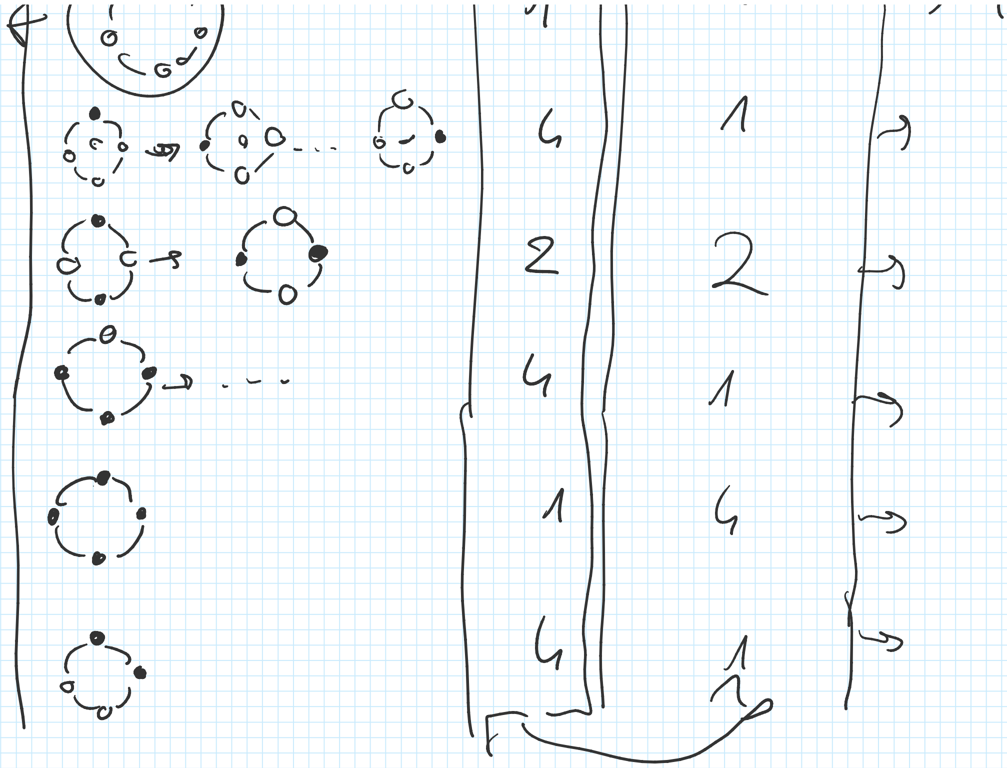


• 90°



(X, R) 90° orbite → R # orbita # stabilizzatore $|G|$

 1 4 → 4

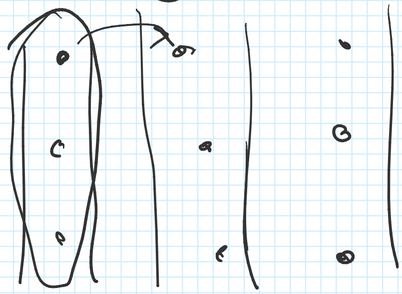


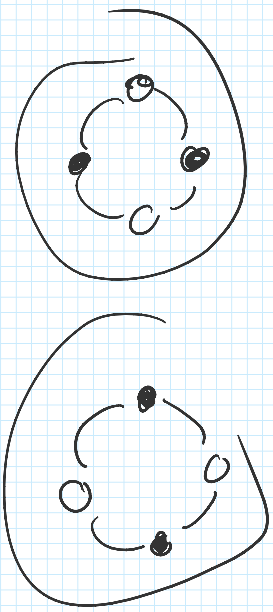
Stabilizzatore: $x \in X \rightarrow \{g \in G : g \circ x = x\}$

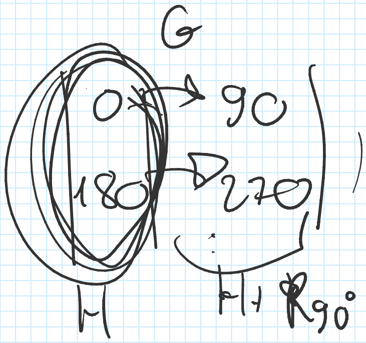
$g(x) = x$

↓

sottogruppo di G







$$\#\text{Orbite} = \sum_{x \in X} \frac{1}{|Orb(x)|} = \sum_{x \in X} \frac{|Stab\, x|}{|G|} =$$

$$\frac{1}{|G|} \underbrace{\sum_{x \in X} |Stab\, x|}_{\left|\left\{(g,x) \,\middle|\, \underset{g \cdot x}{g(x)} = x\right\}\right|,\ g \in G,\ x \in X} = \frac{1}{|G|} \sum_{g \in G} |Fix(g)|$$

fissati con g

$$|Fix(g)| = \left|\{x \in X : g(x) = x\}\right|$$

$$\#\text{Orbite} = \frac{1}{|G|} \sum_{g \in G} |Fix(g)|$$

$R_{90°} \to |Fix(R_{90°})| = 2$

$R_{180°} \to |Fix(R_{180°})| = 4$

$R_{270°} \to |Fix(R_{270°})| = 2$

$R_{0°} \to |Fix(R_{0°})| = 16$

$$\frac{1}{4}(2+4+2+16) = \frac{24}{4} = 6$$

1) Collana con 6 palline e $K \geq 1$ colori. Quante sono le colorazioni distinte a meno di rotazioni?

2) Collana con 5 palle e $K \geq 1$ colori. Quante sono le colorazioni distinte a meno di rotazioni e riflessioni?

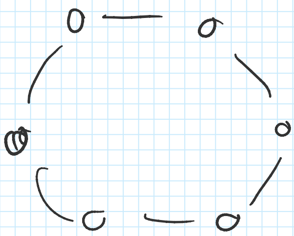
3) Due caselle di scacchiera 11×11 sono dipinte di nero e le rimanenti

3) Due caselle di scacchiera 11×11 sono dipinte di rosso e le rimanenti di blu. Quante colorazioni a meno di rotazioni?

4) Quanti modi di colorare le facce di un tetraedro con k colori, a meno di rotazioni?

$|G| = 6$

1)



$R_{60°} = R_{300°} \to |Fix| = k$

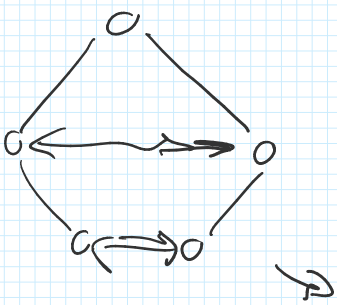
$R_{120} = R_{240°} \to |Fix| = k^2$

R_{180} $|Fix| = k^3$

k^6

$$\frac{k^6 + k^3 + 2k^2 + 2k}{6}$$

2)



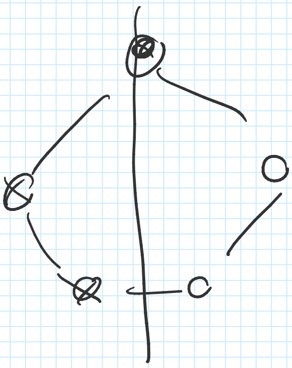
$|G| = 10$

$72°$

$R_1 \to k$

$R_2 = R_3 = R_4$

Id $\to k^5$



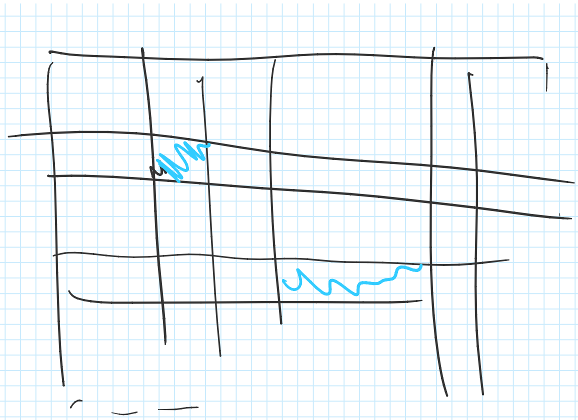
k^3

$$\frac{k^5 + 5k^3 + 4k}{10}$$

3)

$|G| = 4$

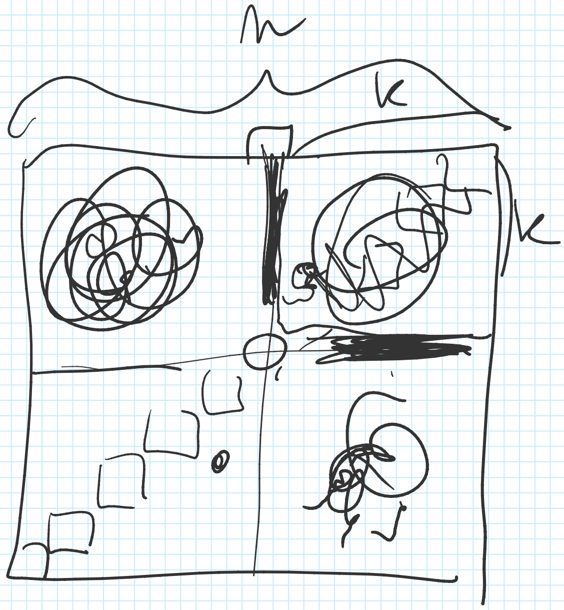
3)



$|G| = 4$

$$\binom{121}{2} = 7260$$

$2k+1 = n$ dispari



id $\binom{n^2}{2}$

Rot 90° $\quad 0$

Rot 270°

Rot 180°

$2k^2 + 2k$

$$\frac{1}{4}\left(\binom{(2k+1)^2}{2} + 2k^2 + 2k\right)$$